TABLE I

Plate Number	Date	Julian Date	V	Standard Error
		2439000+		
E 9393	1965 Nov. 30/Dec. 1	095·4	14·38	±0·02
9396	Nov. 30/Dec. 1	095·6	14·31	0·02
9407	Dec. 2/3	097·6	14·33	0·02
9461	Dec. 20/21	115·6	14·56	0·03
9501	1965 Dec. 28/29	123·4	14·66	0·02
9603	1966 Mar. 20/21	205·3	>15	—
9982	Aug. 17/18	355·6	>16	—
10117	Sept. 17/18	386·7	14·79	0·03
10130	Sept. 19/20	388·7	14·65	0·02
10211	Oct. 20/21	419·6	14·01	0·03
10235	Nov. 9/10	439·5	14·01	0·03
10255	Nov. 14/15	444·6	14·11	0·03
10260	Nov. 15/16	445·6	14·11	0·04
10286	Dec. 2/3	462·6	14·24	0·04
10318	Dec. 6/7	466·6	14·22	0·07
10345	Dec. 8/9	468·6	14·30	0·03
10361	Dec. 10/11	470·5	14·26	0·04
10373	1966 Dec. 16/17	476·5	14·32	0·06
10437	1967 Jan. 3/4	494·5	14·42	0·04
10445	Jan. 12/13	503·5	14·54	0·07
10487	Feb. 4/5	526·3	14·78	0·01
10518	Feb. 8/9	530·4	14·78	0·02
10597	1967 Mar. 5/6	555·4	~15	—

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References

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CORRESPONDENCE

To the Editors of 'The Observatory'

Saturation in Fraunhofer Lines

GENTLEMEN,—

Saturation in a Fraunhofer line means that with increasing absorption the depression in the line (line depth) is no longer proportional to the optical thickness of the absorbing layer, as in the case of weak absorption. In the solar spectrum (intensity) this effect is usually accounted for by Pecker's saturation function[1] based on Unsöld's weighting function theory. In a series of papers, Mugglestone and his co-workers[2,3,4] have recently discussed the saturation effect in a way comparable with Pecker's idea. Both methods are subject to the disadvantage that the concept of a saturation function means a manipulation of the weak line theory, instead of finding a general theory including the weak absorption as a first-order approximation. Moreover, stellar spectra (flux) cannot be exactly described by saturation functions at all.

Using Mugglestone's original ideas, the concept of a saturation function can be completely abandoned[5]. If we confine our attention to pure absorption, the line depth $r_\lambda(\mu)=(I_0(\mu)-I_\lambda(\mu))/I_0(\mu)$ can be written in the solar case

$$r_\lambda(\mu) = \int_0^\infty g(\tau , \mu)\, \phi\left(v ; \frac{\tau_c}{\mu} , a\right) \mathrm{d}\tau \qquad (1)$$

[I_0 continuous intensity; I_λ line intensity; arc cos μ angle between the normal to the surface and the direction of observation], where $g(\tau , \mu)$ is a modified weighting function

$$g(\tau , \mu) = \frac{1}{I_0(\mu)} \frac{\mathrm{d}B}{\mathrm{d}\tau} e^{-\frac{\tau}{\mu}} \qquad (2)$$

[$B(\tau)$ Kirchhoff-Planck-function, τ continuous optical depth], and $\phi\left(v ; \frac{\tau_c}{\mu} , a\right)$ is a "function of growth",

$$\phi\left(v ; \frac{\tau_c}{\mu} , a\right) = 1-e^{-\frac{\tau_c}{\mu} H(a, v)} \qquad (3)$$

being the line depth in an absorption tube of optical thickness τ_c/μ. [$H(a , v)$ Voigt-function; a damping parameter; distance from line centre $v=\Delta\lambda/\Delta\lambda_D$; $\Delta\lambda_D$ Doppler-width; τ_c line optical depth in the line centre.] a is assumed to be constant with depth.

In the stellar case the above formulae are:

$$R_\lambda = \int_0^\infty G(\tau)\ \Phi(v ; \tau , \tau_c , a)\ \mathrm{d}\tau \qquad (4)$$

with the weighting function

$$G(\tau) = \frac{2}{F_0} \frac{\mathrm{d}B}{\mathrm{d}\tau} E_3(\tau) \qquad (5)$$

[F_0 continuous flux; $E_3(\tau)$ integro-exponential function] and the function of growth

$$\Phi(v ; \tau , \tau_c , a) = 1-E_3(\tau+\tau_c H(a , v))/E_3(\tau) . \qquad (6)$$

The physical meaning of equations (1) and (4) is obvious: the line profile in a solar (stellar) atmosphere is a weighted sum of absorption tube profiles, the weight containing the re-emission. The qualities of the line only enter the functions of growth. The latter ones are shown in Fig. 1 for some selected distances from the line centre v, a set of values of τ, and the damping parameter $a=0{\cdot}1$. The function $\phi(v ; \tau_c , a)$ is identical with $\Phi(v ; \tau = \infty , \tau_c , a)$.

The curves may be interpreted as follows: as long as the absorption is weak, Φ varies linearly with the optical thickness of the absorbing slab τ_c.

With increasing τ_c saturation sets in, and the curves finally converge towards unity (complete saturation).

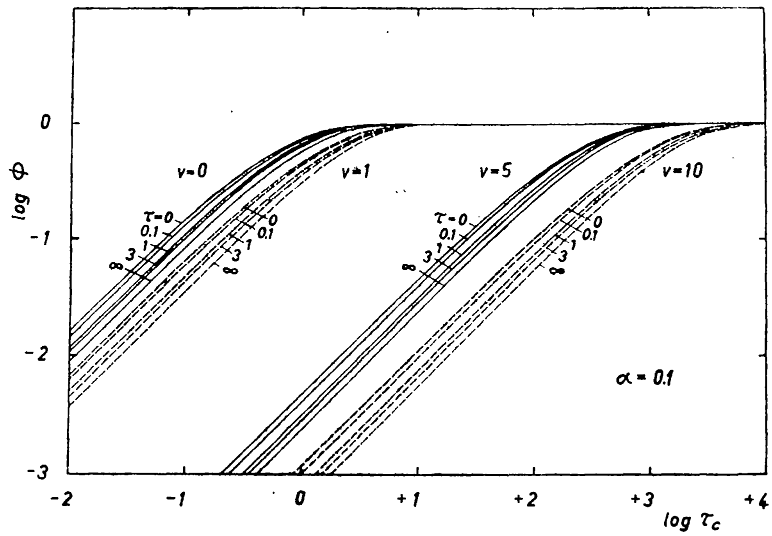

FIG. 1

Functions of growth (line profile).

The equivalent width of a line is obtained by integrating the line profile over all frequencies. On assuming the line to be symmetrical, and the Doppler width to be constant with depth, from equation (1) and (4) follow

$$\frac{w(\mu)}{2\Delta\lambda_D} = \int_0^\infty g(\tau, \mu)\, \psi\left(\frac{\tau_c}{\mu}, a\right) d\tau \quad \text{(solar case)}, \tag{7}$$

$$\frac{W}{2\Delta\lambda_D} = \int_0^\infty G(\tau)\, \Psi(\tau, \tau_c, a)\, d\tau \quad \text{(stellar case)}. \tag{8}$$

The functions of growth (with respect to equivalent width) are the equivalent widths in an absorption tube:

$$\psi\left(\frac{\tau_c}{\mu}, a\right) = \int_0^\infty \phi\left(v ; \frac{\tau_c}{\mu}, a\right) dv\ , \tag{9}$$

$$\Psi(\tau, \tau_c, a) = \int_0^\infty \Phi(v ; \tau, \tau_c, a)\, dv\ . \tag{10}$$

Their interpretation may be deduced from Fig. 2. With increasing optical thickness of the absorbing slab τ_c the curves deviate from linearity: saturation begins. The absorption takes place mainly in the core of the line. At larger thicknesses τ_c damping sets in; the line begins to absorb in the wings: saturation is reduced.

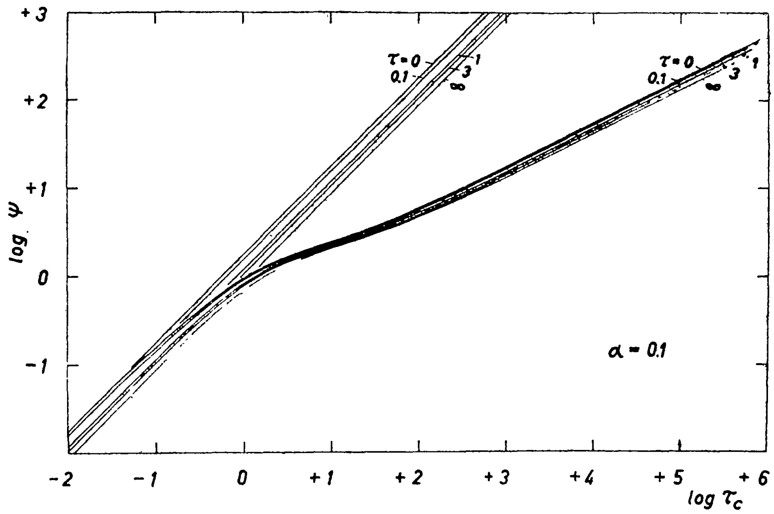

FIG. 2
Functions of growth (equivalent width).

This behaviour is well-known from the curves of growth. Indeed, the curve $\Psi(\tau = \infty, \tau_c, a) = \psi(\tau_c, a)$ is van der Held's curve of growth[6] in an absorption tube. Besides, taking an appropriate mean value C instead of τ_c, the functions of growth can be used as curves of growth in the form[5]

$$\frac{1}{2\Delta\lambda_D}\frac{w}{r_c} = \psi(C, a) \qquad (11)$$

with the limiting line depth

$$r_c = \int_0^\infty \left\{\frac{g(\tau, \mu)}{G(\tau)}\right\} d\tau \, . \qquad (12)$$

The results will be essentially unaltered if the depth dependence of the Doppler width and the damping constant is taken into account.

I am, Gentlemen,
Yours faithfully,
E. A. GUSSMANN

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Potsdam.

1967 February 23.

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